Chapter 3A

Shear

Addenda to Shear #3.1.1

The main difference between the AS3600-2009 and AS3600-2018 Standard is the use of Modified Compression Field Theory [MCFT] to analyze beams in shear.

The design shear strength of a beam is still ϕV_u where the ultimate shear strength V_u is made up of a beam (concrete) component V_{uc} and a shear reinforcing component V_{us}. However the method by which the V_{uc} component is worked out is completely different to the previous Standard. Having said that, many academics have informed the writer that the answers obtained from either method are not that different (when using the same ϕ factor).

It must be noted here that when the AS3600 Standard was updated in 2018, they also modified (ie relaxed) all the ϕ factors. Thus where ϕ = 0.70 for shear in the 2009 Standard, it was increased to 0.75 (about 7% increase). This effectively means that the shear capacity of beams now is slightly higher than it would have been using the older version of the Standard (purely based on the ϕ factor). Note in the formula below, P_v is the upward component of any prestress.

$$V_u = V_{uc} + V_{us} + P_v$$ ...Equation 3.1A

Addenda to 3.1.2 Shear Strength V_{uc} of Unreinforced Concrete Beam

The main focus of the concrete shear strength in the 2018 Standard is longitudinal strain ε_x in the main reinforcement.

Aggregate size and depth of member still play a role in the shear resistance of beams but the key factor is longitudinal strain.

As in the previous version of AS3600, the 2018 version limits the concrete shear capacity $\sqrt{f'_c}$ to a maximum of 8.0 *MPa* (which is equivalent to saying that no additional shear strength can be gained by using concretes whose f'_c > 65 *MPa*).

The magnitude of V_{uc} which takes into account all these components is calculated by an empirical formula based on experimental data and it is given by Equation 3.2A.

$$V_{uc} = k_v b_v d_v \sqrt{f'_c}$$ ...Equation 3.2A

where:

b_v = effective shear web width

d_v = effective shear depth (taken as greater of 0.72D or 0.9d)

$$When\ A_{sv} < A_{sv.\min}$$

$$k_v = \left(\frac{0.4}{1+1500\varepsilon_x}\right).\left(\frac{1300}{1000+k_{dg}d_v}\right)$$

$$where$$

$$d_v = 0.72D\ or\ 0.90d$$

$$k_{dg} = \left[\frac{32}{16+d_g}\right] but \geq 0.8$$

$$or = 1.0\ (if\ 20\ mm\ \max.\text{size}\ aggregate\ used)$$

$$When\ A_{sv} > A_{sv.\min}$$

$$k_v = \left[\frac{0.4}{1+1500\varepsilon_x}\right]$$

$$Longitudinal\ Strain\ in\ Concrete\ \varepsilon_x\ for\ Shear$$

$$\varepsilon_x = \frac{|M^*/d_v| + |V^*| - P_v - A_{pt}f_{po}}{2\left(E_sA_{st} + E_pA_{pt}\right)} \quad \ldots \quad but \leq 0.003$$

Once you remove the prestress components from the formula, it simplifies to the following expression:

$$Longitudinal\ Strain\ in\ Reinforced\ Concrete\ \varepsilon_x\ for\ Shear$$

$$\varepsilon_x = \frac{|M^*/d_v| + |V^*|}{2\left(E_sA_{st}\right)} \quad \ldots \quad but \leq 0.003$$

The angle of inclination θ_v of the concrete compression strut to the longitudinal axis of the member (and thus the approximate angle at which shear cracks form) is given by the formula

$$\theta_v = \left(29^o + 7000\varepsilon_x\right)$$

3.1.3 Maximum Ultimate Shear Strength $V_{u.max}$

The maximum shear force formula in the AS3600-2018 Standard looks quite different to the AS3600-2009 version however once you simplify it you will see it is very similar to the 2009 formula.

The $V_{u.max}$ (# 8.2.6) formula in the 2009 version which causes crushing of the concrete was given by

$$V_{u.\max} = 0.2 f_c^{'} b_v d_o \quad \text{...Equation 3.9}$$

The AS3600-2018 version is expressed in this manner:

$$V_{u.\max} = 0.55\left[f_c^{'} b_v d_v \left(\frac{\cot\theta_v + \cot\alpha_v}{1+\cot^2\theta_v} \right) \right] + \mathrm{P_v}$$

However, few engineers incline their stirrups at an angle other than straight downwards thus α_v = 90 deg. The expression in the brackets then simplifies to: 1 / (tan θ_v + cot θ_v). Ignoring the prestress component P_v the $V_{u.max}$ expression simplifies to:

$$V_{u.\max} = 0.55 f_c^{'} b_v d_v \left(\frac{1}{\tan\theta_v + \cot\theta_v} \right)$$

If we assume a lower bound value of θ_v of 30 degrees then the expression in the brackets simplifies to 0.433 which when multiplied by 0.55 gives 0.23. If you then replace d_v with $0.9d_o$ then you arrive at $0.21f'_c b_v d_o$ which is very close to the original AS3600-2009 formula.

3.1.6A Min Shear Reinforcement $A_{sv.min}$ and Min Shear Strength $V_{u.min}$

The #8.2.8 formula in AS3600-2009 for minimum shear area in beams used to contain the coefficient 0.06 but it has now been increased to 0.08 as per AS3600-2018 #8.2.1.7 (ie a 33% increase). Similarly, the second component of the #8.2.8 2009 formula ie $0.35b_v s/f_{sy.s}$ has also been removed.

$$A_{sv.\min} = \left(\frac{0.08\sqrt{f'_c}\, b_v\, s}{f_{sy.f}} \right) \quad \text{... Equation 3.10A}$$

If the size of the shear reinforcement, area A_{sv} is chosen, the maximum spacing s to satisfy Equation 3.10 is obtained by transposing the equation for s as shown in Equation 3.11.

$$s \le \left(\frac{A_{sv} f_{sy.f}}{0.08\sqrt{f'_c}\, b_v} \right) \quad \text{...Equation 3.11A}$$

It must be noted that the spacing given by Equation 3.11A is often larger than the maximum stirrup spacing given in the Standard (which is usually 300 *mm*).

In AS3600-2018 #8.3.2.2, it states "*In members not greater than 1.2 metres in depth, the maximum longitudinal spacing shall not exceed the lesser of 300 mm and 0.5D; otherwise, ... 600 mm. The maximum transverse spacing across the width of the member shall not exceed the lesser of 600 mm and D"*.

The minimum shear strength # 8.2.9 $V_{u.min}$ in AS3600-2009 has also been removed from the beam section in the AS3600-2018 Standard. Similarly all the AS3600-2009 #8.2.5 rules regarding if $0.5\phi V_{uc} \le V^* \le 0.5\phi V_{u\ min}$, then minimum shear reinforcement being required have also been removed from AS3600-2018.

3.1.8A Contribution to Shear Capacity by Vertical Stirrups

The contribution provided by shear reinforcement in the form of vertical stirrups is simply the tensile capacity of the number of vertical stirrups contained within a potential shear crack inclined at an angle θ_v.

Assuming that the shear angle θ_v is fixed, the number of stirrups n crossing a potential crack shown in Figure 3.8 is given by the formula below where z is the lever arm of the internal couple

$$n = \frac{z \, Cot \, \theta_v}{s}$$

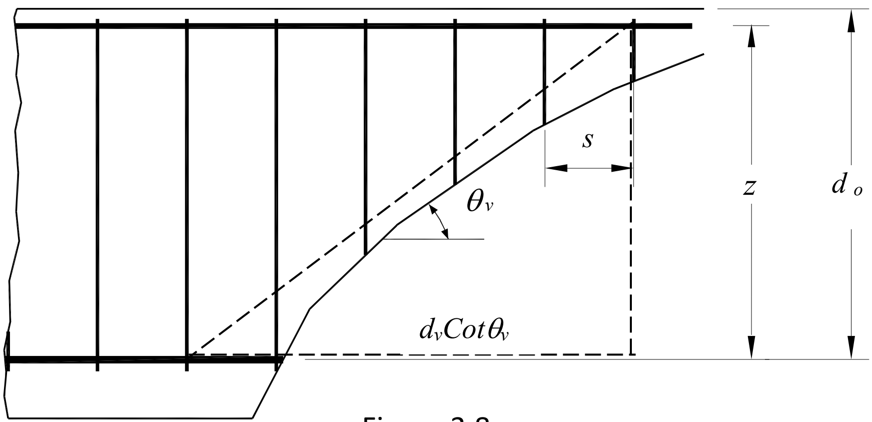


Figure 3.8

The average value of z is about $0.9d_o$. and in AS3600-2018 is now referred to as d_v.

$$n = \frac{d_v \, Cot \, \theta_v}{s}$$ ...Equation 3.13A

Ultimate tensile strength V_{us} of all the stirrups intersected by the diagonal tension crack is:

$$V_{us} = n\, A_{sv}\, f_{sy.f} \qquad \text{...Equation 3.14}$$

where

A_{sv} = Shear area, which is the area of two (2) vertical legs of the stirrups or fitments
$f_{sy.f}$ = Yield strength of stirrups or fitments

The effective tensile capacity of the stirrups is ϕV_{us} is given by Equation 3.15A which is derived from Equation 3.14 where Equation 3.13A has been substituted for n. Note in the AS3600-2018 Standard the only difference between the previous version and the new one here is the substitution of d_o with d_v.

$$V_{us} = \frac{A_{sv} f_{sy.f}\, d_v Cot\,\theta_v}{s} \qquad \text{...Equation 3.15A}$$

The shear steel requirement is thus given as the required stirrup spacing to carry the excess shear which is equal to ($\phi\, V_{us}\, required = V^* - \phi\, V_{uc}$). Alternatively substituting ($V^* - \phi\, V_{uc}$) for $\phi\, V_{us}$ in Equation 3.15A, we now provide an expression for the maximum tie (ie stirrup or fitment) spacing s using Equation 3.17A below

$$s \le \frac{\phi\, A_{sv}\, f_{sy,f}\, d_v}{\left(V^* - \phi V_{uc}\right) \tan\theta_v} \qquad \text{...Equation 3.17A}$$

In view of on-site potential inaccuracies and uncertainties it is recommended that closed shear reinforcement should be used in preference to the open U-shaped shear reinforcement because it provides better anchorage, a much more rigid reinforcing cage securing the longitudinal reinforcement and it is effective in resisting torsion. (Note remember that # s_{max} is usually 300mm)

With regards to combined shear and torsion, AS3600-2018 combines the two actions into a single formula. Note however that if applied design torsion is zero then the value for V^*_{eq} merely becomes the design shear V^*.

$$V^*_{eq} \le \sqrt{\left(V^*\right)^2 + \left(\frac{0.9T^* u_h}{2A_o}\right)^2}$$

Example 3.1A

Beam section in Figure 3.6 uses N50 concrete.

The beam is required to carry a design shear force $V^* = 240$ kN and design moment near the support of M* = 46 kNm

➔ Determine the following parameters:

(a) Longitudinal strain ε_x
(b) Shear Angle θ_v
(c) Web shear crushing capacity $V_{u.max}$
(d) Concrete Shear Capacity V_{uc}
(e) Shear reinforcement requirement V_{us}

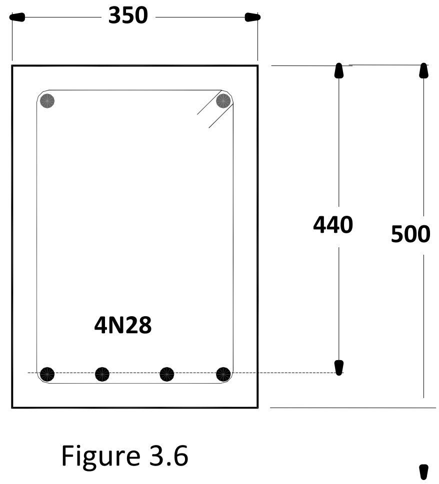
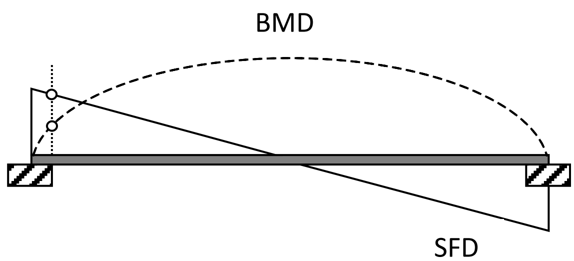


Figure 3.6

<u>Solution</u>

Data:
$E_s = 200{,}000$ MPa
A_{st} = 4-N28
$= 2460\ mm^2$
$b = 350\ mm$
$d = 440\ mm$



$$d_v = 0.72D \text{ or } 0.90d \text{ (greater)}$$
$$= 0.72(500) \text{ or } 0.90(440)$$
$$= 360\ mm \text{ or } 396\ mm$$
$$= 396\ mm$$
$$k_{dg} = 1.0 \text{ (as max.agg} = 20\,\text{mm)}$$

$p = A_{st}/bd$
$= 2460/(350 \times 440)$
$= 0.016$ (ie 1.6%)
p_{max} (50 MPa) = 2.39% thus OK (since $0.016 < 0.0239$)

$V^* = 280\ kN$
$M^* = 46\ kNm$ (near support where V* taken)

(a) Longitudinal shear strain ε_x

$$\varepsilon_x = \frac{|M^*/d_v| + |V^*|}{2(E_s A_{st})} \quad \ldots \quad but \le 0.003$$

$$= \frac{|46E6/396| + |240E3|}{2(200,000 \times 2460)}$$

$$= 0.00036 \qquad \le 0.003 \therefore OK$$

(b) Shear angle θ_v

$$\theta_v = (29^o + 7000\varepsilon_x)$$

$$= (29^o + 7000 \times 0.00036)$$

$$= 31.5^o$$

(c) Maximum shear strength for web shear crushing.

Using the modified AS3600-2018 version of $V_{u.max}$

$$V_{u.max} = 0.55 f_c' b_v d_v \left(\frac{1}{\tan\theta_v + \cot\theta_v} \right)$$

$$= 0.55 \times f_c' b_v d_v \left(\frac{1}{\tan 31.5 + \cot 31.5} \right)$$

$$= 0.55 \times f_c' b_v d_v (0.4455)$$

$$= 0.245 \times 50 \times 350 \times 396 / 1000$$

$$= 1697\,kN$$

$$\phi V_{u.max} = 0.75 \times 1697\,kN$$

$$= 1273\,kN \quad \therefore OK$$

$$as \;\; V^*(= 240\,kN) < \phi V_{u.max} (= 1273\,kN)$$

(d) Concrete Shear Capacity V_{uc}

For the concrete section alone (assuming no shear steel and max aggregate size is 20 mm)

$\phi V_{uc} = k_v\, b_v\, d_v\, \sqrt{f'_c}$

where

$$\begin{aligned}
\varepsilon_x &= 0.00036 \\
d_v &= 0.72D \text{ or } 0.90d \ (\text{greater}) \\
&= 0.72(500) \text{ or } 0.90(440) \\
&= 360\ mm \text{ or } 396\ mm \\
&= 396\ mm \\
k_{dg} &= 1.0
\end{aligned}$$

When $A_{sv} < A_{sv.\min}$

$$\begin{aligned}
k_v &= \left(\frac{0.4}{1+1500\varepsilon_x}\right).\left(\frac{1300}{1000+k_{dg}d_v}\right) \\
&= \left(\frac{0.4}{1+1500\times 0.00036}\right).\left(\frac{1300}{1000+1.0\times 396}\right) \\
&= \left(\frac{0.4}{1+0.54}\right).\times\left(\frac{1300}{1396}\right) \\
&= (0.26)\times(0.93) \\
&= 0.24
\end{aligned}$$

$$\begin{aligned}
V_{uc} &= k_v b_v d_v \sqrt{f'_c} \\
&= 0.24\times 350\times 396\times\sqrt{50} \quad /1000 \\
&= 235\, kN
\end{aligned}$$

$$\begin{aligned}
\phi V_{uc} &= 0.75\times 235 \\
&= 176\ kN
\end{aligned}$$

(e) Shear Steel Shear Capacity V_{us}

Since 176 kN < 240 kN, shear reinforcement will therefore be required to carry the difference between ϕV_{uc} and V^* ie $\phi V_{us} = V^* - \phi V_{uc}$.

Deciding to use N12 stirrups, and using the revised $A_{sv.min}$ expression, where stirrup spacing s was made the subject. Note since shear reinforcement is usually U-shaped, then two stirrup legs are included in the calculations for A_{sv}.

$$s_{max} \le \left(\frac{A_{sv} f_{sy.f}}{0.08 b_v \sqrt{f_c^{'}}} \right)$$
$$\le \frac{2 \times 110 \times 500}{0.08 \times 350 \times \sqrt{50}}$$
$$\le 555\ mm$$

This exceeds the usual maximum stirrup spacing of 300 mm thus adopting this value, we will see if we satisfy the ϕV_{us} value.

Assuming N12 @ 300 mm spacing, and that the required ϕV_{us} is given by

$$\phi V_{us} = V^* - \phi V_{uc}$$
$$= 240 - 176$$
$$= 64 \text{ kN}$$

$$V_{us} = \frac{A_{sv} f_{sy.f}\, d_v Cot\, \theta_v}{s}$$
$$= \frac{220 \times 500 \times 396 \times Cot\, 31.5}{300} / 1000$$
$$= 236 \text{ kN}$$

$$\phi V_{us} = 0.75 \times 236$$
$$= 177\, kN \quad \ldots > 64\, kN \therefore OK$$

It can be then that N12 stirrups at 300mm spacing definitely satisfied the requirement.

Example 3.3A

For the reinforced concrete beam shown in Figure 3.12, calculate the following parameters (assuming the applied design shear force V* = 280 kN and moment at end is zero).

(a) Maximum permissible shear force $V_{u.max}$

(b) Unreinforced beam shear capacity ϕV_{uc}

(c) Spacing of N12 stirrups at a section where the design shear force $V^* = 280$ kN

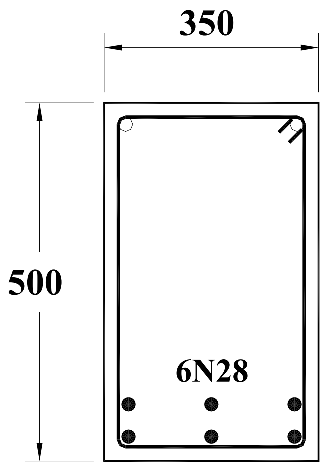


Figure 3.12

Solution

Data: $b_v = 350$ mm $D = 500$ mm $A_{st} = 3720$ mm^2 $A_{sv} = 220$ mm^2

$V^* = 280$ kN $f'_c = 32$ MPa $f_{sy.f} = 500$ MPa

Exposure A2 Minimum cover to shear reinforcement = 25mm

➔ Check p vs $p\%_{max}$ steel limit

$$p = \frac{A_{st}}{b_v d_o}$$

$$= \frac{3720}{350 \times 449}$$

$$= 0.0237$$

$$(ie\ 2.37\%)$$

Effective depth d_o = D – cover – stirrup – half bar
= 500 – 25 – 12 – 0.5 × 28
= 449 mm

Shear depth d_v = $0.9d$ or $0.72D$ (greater of)
= 0.9 ×449 or 0.72 × 500
= 404 mm or 360 mm
= 404 mm

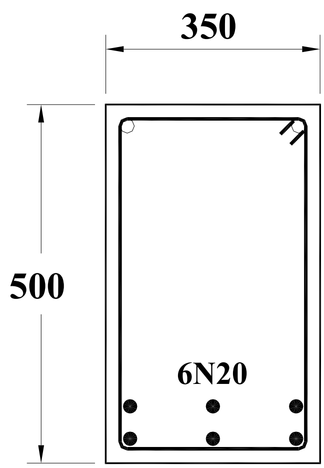


Figure 3.12A

$$p = k_u \alpha_2 . \gamma \left(\frac{f'_c}{f_{sy}} \right)$$

$$p_{\max} = 0.36 \times 0.80 \times 0.89 \left(\frac{32}{500} \right)$$

$$= 0.0164$$

$$(> p = 0.0237! \quad \rightarrow \textit{thus redesign beam})$$

$p_{\max}$ = 0.0164 × 350 × 449
= 2577 mm^2
(ie 6-N20 bars giving A_{st} = 1884 mm^2 ie 1.2% – see Figure 3.12A)

Now calculate shear properties based upon a grade 32MPa concrete beam measuring 350 mm wide × 500 mm overall depth with 6-N20 bars.

Longitudinal shear strain ε_x

$$\varepsilon_x = \frac{|M^*/d_v| + |V^*|}{2(E_s A_{st})} \quad \ldots \quad but \leq 0.003$$

$$= \frac{|0/404| + |280E3|}{2(200{,}000 \times 2577)}$$

$$= 0.00037 \qquad \leq 0.003 \therefore OK$$

Shear angle θ_v

$$\theta_v = (29^o + 7000\varepsilon_x)$$

$$= (29^o + 7000 \times 0.00037)$$

$$= 31.6^o$$

(a) Maximum shear strength for web shear crushing.

Using the modified AS3600-2018 version of $V_{u.max}$

$$
\begin{aligned}
V_{u.max} &= 0.55 f_c' b_v d_v \left(\frac{1}{\tan\theta_v + \cot\theta_v} \right) \\
&= 0.55 \times f_c' b_v d_v \left(\frac{1}{\tan 31.6 + \cot 31.6} \right) \\
&= 0.55 \times f_c' b_v d_v (0.446) \\
&= 0.245 \times 32 \times 350 \times 404 / 1000 \\
&= 1100\, kN
\end{aligned}
$$

$$
\begin{aligned}
\phi V_{u.max} &= 0.75 \times 1100\, kN \\
&= 833\, kN \quad \therefore OK
\end{aligned}
$$

$$as \;\; V^* (= 280\, kN) < \phi V_{u.max} (= 833\, kN)$$

(b) Unreinforced beam shear capacity ϕV_{uc}

For the concrete section alone (assuming no shear steel and max aggregate size is 20 mm)

$\phi V_{uc} = k_v\, b_v\, d_v \sqrt{f'_c}$

where

$$
\begin{aligned}
\varepsilon_x &= 0.00037 \\
d_v &= 0.72D \text{ or } 0.90d \text{ (greater)} \\
&= 0.72(500) \text{ or } 0.90(449) \\
&= 360\, mm \text{ or } 404\, mm \\
&= 404\, mm \\
k_{dg} &= 1.0
\end{aligned}
$$

When $A_{sv} < A_{sv.\min}$

$$
\begin{aligned}
k_v &= \left(\frac{0.4}{1+1500\varepsilon_x}\right).\left(\frac{1300}{1000+k_{dg}d_v}\right)\\
&= \left(\frac{0.4}{1+1500\times 0.00037}\right).\left(\frac{1300}{1000+1.0\times 404}\right)\\
&= \left(\frac{0.4}{1+0.54}\right).\times\left(\frac{1300}{1404}\right)\\
&= (0.26)\times(0.93)\\
&= 0.24
\end{aligned}
$$

$$
\begin{aligned}
V_{uc} &= k_v b_v d_v \sqrt{f'_c}\\
&= 0.24\times 350\times 404\times\sqrt{32} \quad /1000\\
&= 192\,kN
\end{aligned}
$$

$$
\begin{aligned}
\phi V_{uc} &= 0.75\times 192\\
&= 144\ kN
\end{aligned}
$$

The design shear force $V^* = 280$ kN exceeds the minimum shear capacity $\phi V_{u.min}$ hence shear reinforcement will be required for the excess shear force $V^* - \phi V_{uc}$

Required shear capacity of the shear reinforcement

$$
\begin{aligned}
\phi V_{us} &= V^* - \phi V_{uc}\\
&= 280 - 144\\
&= 136 \text{ kN}
\end{aligned}
$$

Using the formula derived earlier we can determine the stirrup spacing s to satisfy the extra capacity to come from the shear steel. Again using N12 stirrups, the spacing would be:

(c) Stirrup Spacing s

$$
\begin{aligned}
s &\le \frac{\phi\, A_{sv}\, f_{sy,f}\, d_v}{\left(V^* - \phi V_{uc}\right)\tan\theta_v} \\
&\le \frac{0.75 \times 220 \times 500 \times 404}{136E3 \times \tan 31.6^o} / 1000 \\
&\le 398\ mm
\end{aligned}
$$

thus use

$s_{\max} = 300\ mm$